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MATHEMATICAL SYMBOLISM AND THE LEARNING OF MATHEMATICS: SOME QUESTIONS.

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This paper will briefly discuss two aspects of the use and learning of mathematical symbolism.

One of them is the nature of the difficulties with algebraic symbolism and the other one is about the different roles that mathematical symbolism can assume.

I'll begin with a very well known item:

" *if* $e+f=8$, *then* $e+f+g=$ ____" (from CSMS algebra project)

Kuchemann (1978, p.25) analizes data from a third year group (from CSMS project) and observes that the high percentage of failure (59%) could be explained using the concept of "Acceptance of Lack of Closure" (ALC).

Biggs and Collis (1982, p. 67) consider that "*the development in children's thinking and responding, both mathematically and otherwise, can be traced in terms of the need to close or to come to a definite decision*", and proceed to show how this can be used to characterize levels of thinking and responding within the arithmetical-algebraic domain.

Nevertheless, when talking of a more general context, Biggs and Collis (p. 27) characterize closure as "the need to come to a conclusion <u>of some kind</u> (to close)" (my emphasis).

Moreover, Booth (1984, p. 91) points out to the fact that "the apparent effectiveness of the teaching programme in restructuring childrens' thinking in this regard would suggest that the notion [ALC] was not beyond the conceptual grasp of these children". He was talking about children 12 to 15 years-old.

At this point, I will formulate my first question:

TO WHAT EXTENT IT'S USEFUL, WHEN INVESTIGATING THE LEARNING OF ALGEBRAIC SYMBOLISM, TO PUT THE STRESS ON THE EXPECTED COGNITIVE DEMAND? WHICH OTHER ASPECTS SHOULD BE BROUGHT TO A COMPREHENSIVE "PALETTE"?

The more general formulation of <u>closure</u> , quoted above, gives us the opportunity to reinterpet its application to algebraic symbolism.

Instead of thinking of "<u>the</u> need to come to a conclusion" , we should ask ourselves: "What is an acceptable definition of conclusion? How do we know we 'came to a conclusion'?". Any answer will depend on what my objectives are. In other words, it depends on <u>what</u> is my problem.

Let's examine another example.

Consider the question: " 3x5 + 8 - 37 = ..." , to be solved within the set of natural numbers.

Solution: 3x5 + 8 - 37 =
15 + 8 - 37 =
23 - 37

Answer : It's not possible to solve it within N (because if a<b, then a-b is not a natural number)

Why not to give '23 - 37' as the answer? Simply because the problem was to <u>evaluate</u> the expression and not to simplify it as it was the case with the CSMS item. (Would "e+f+g" be an answer ?)

The fact that after the teaching programme significant gains were achieved on items related to re-writing or re-presenting (Booth, 1234, p. 74 - a report of CSMS project), is a support to the hypothesis that the main difficulty is related to the question "What am I supposed to do?" rather then to the alternative question "How do I do it?".

In other words, it's not a matter of cognitive competence but a matter of misguided performance. And we may well say that our performance is to a large extent guided by the perceived purpose of the task.

My third example deals with this question.

The "students-and-professors problem" (S-P) has been thoroughly analysed:

"Write an equation using the variables S and P to represent the following statement: 'There are six times as many students as professors in this university.' Use S for the number of students and P for the number of professors."

The first approach (see, for example:Clement, 1982; Rosnick, 1981) was to credit the "reversal error" (writing 6S=P, which accounted for most of the 37% of errors) to a lack of understanding of the concept of variables. Rosnick suggested that "*it might even be the case that many secondary school students and, for that matter, college students have not yet reached the necessary level of intelectual development to be able to do that distinction.*"

Wollman (1983) contested this view on the basis that "reversal subjects" had no difficulties in extracting and manipulating information from both the verbal statement and expressions like y=6x . He concluded that the difficulty was in the translation process and devised some sucssesful teaching strategies to overcome it. But he was still concerned with the error.

One question was left to be asked: "How powerful was that reversed representation to the subjects that 'adopted' it?"

My attempt to explore this question took the form of an investigation (still in progress) where subjects (university students) are asked to determine whether some given items (algebraic statements/mixed statements/tables) are or not in agreement with the situation expressed by a given verbal statement and to present a justification for the option taken.

One of the statements used is the one in the **S-P** problem. The fact that it could induce reversal errors (by the use of inconsistent language) is, in this study, an advantage rather then a problem.

The items presented to validation were:

(a) $P = 6S$ (b) $\frac{S}{6} = P$ (c) $S = 72$ .... $P = 12$; $S = 24$ .... $P = 4$

(d) (no. of professors) = (no. of students) : 6

(e) $3 \times S = 18 \times P$ (f) $S = 6 \times P$

Subject: a post-graduate student in Politics.

Items (a), (c), (d) were considered to be correct.

The other items were considered to be incorrect for the following reasons:

(b) *"The relationship should be the inverse,* $S = \frac{P}{6}$ *; here we have* 1/6 of *student for one professor. That's not compatible with the initial statement.*

(e) *"No:* $\cancel{3}^{1} \times S = \cancel{18}^{6} \times P$ [Sic] *and the relationship should be the inverse to be true"*

(f) *"No, because the number of students should be 6 xmore the number of professors* [Sic]"

It can be clearly seen that:

1) It was a consistent use of symbols;

2) The subject was able to deal with variables (item d; this was further verified by the easy solution of 'Which is greater: 2n or n+8 ?')

3) The subject applied "internal" transformations to the notation (item e)

The choice was a confident one, not a mistake or a "lower" performance.

I do accept the fact that if the suject had a solid (technical) meaning for the word "equation", most probably the prompting would be attended.

My suggestion is that the choice was made so to answer the question "Represent the situation" rather then to answer the 'correct' question

"Represent the numerical relationship implicit in the situation" (which is actually the prompting in "use an equation").

The succes in Wollman's teaching experiment was actually due to making clear that "you're being asked to produce a formula". From my point-of-view it's not surprising at all to learn that after that they produced a far greater amount of correct responses.

Again we are faced with the "What am I supposed to do here ?" question.

However we must also face a different but even sharper question: **Is it true that by demanding pupils to use mathematical symbolism in the "official fashion" we are improving their ability to use it as a help to solve problems (specially new ones)?**

I think those are questions worth some deeper discussion.

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