Categories of everyday life as elements organising mathematics teacher education and development projects

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In this paper we consider a key aspect of the professional practice of mathematics teachers — acting in urgency and making decisions in uncertain situations —, and propose that this issue can be properly addressed by the addition of two components to the design of mathematics teacher education projects: (i) to foster the teachers' ability to *read* her students *knowledge production* and *meaning production*, in the sense of the Model of Semantic Fields (MSF); and, (ii) to foster teachers' willingness to accept differences in *meaning production*. We argue that this can be better achieved through the adoption of a new set of categories in the organisation of part of those projects (the courses), and propose that categories from everyday life be chosen. On the basis of this choice is the need to have projects that bring together actual needs of practice (particularly productive interaction) and the needs of background preparation — including mathematical maturity and understanding. Divides like pedagogy-mathematics, practice-theory and elementary-advanced mathematics (content to teach vs. foundations) are addressed and re-placed in the overall design. An exemplary example of one such category, together with a brief description of how to proceed within it, is offered.

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Romulo Lins[1]

The professional activity of mathematics teachers involves almost continually acting in urgency and making decisions in uncertain situations (Perrenoud, 1999). This feature of the profession is, or should be, at the centre of our considerations as mathematics teacher educators. But how can one prepare a teacher to deal with situations that are precisely, as we have said, *uncertain*? Providing him or her with a significant repertoire of situations and solutions is useful, but certainly not sufficient, as diversity will probably show up everyday and in many different forms.

In this paper I will argue that two key components in the design of mathematics teacher education projects can properly address this need: (i) to foster the teachers' ability to *read* her students knowledge production and meaning production, in the sense of the Model of Semantic Fields (MSF; as in Lins, 2001). This means that instead of making decisions based on normative, prescriptive, knowledge, she will act upon what is actually being said by students, always aiming at what we call *productive interaction*; and, (ii) to foster teachers' willingness to accept differences in *meaning production* (again, as proposed in the MSF; Lins, 2001).

Point (i) means that teachers should be able to act towards eliciting what are the *objects* the students are talking about or thinking with, and this has to do with the *meanings* they are producing (Lins, 2001; Lins & Garcia Jr., 1995). But point (i) alone could suggest that this is done in order to know, in greater detail than the usual right/wrong, if some action is required to 'correct the wrong,' that is, to suggest some improvement to usual teaching.

Point (ii) adds a component that prevents this view from being naturalised. It is actual *acceptance* of meanings that do not converge with those produced by the teacher for the same 'thing,' that is being proposed here. And the reason for that is not to postpone — as in some form of benevolent teaching — the time to correct what is wrong. When we speak of *productive interaction*, we refer to the possibility that students and teachers be speaking *in the same direction*, so what one says does not seem paradoxical to the other.[2] An example of how this works in the classroom can be found in Lins (in print). [3] What is innovative here is not the fineness of the reading — which is certainly greatly improved by the use of the categories of the MSF —, but the intention

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[2] This could also be framed in terms of 'meaning negotiation'; I prefer, though, to stick to 'productive interaction', both because it refers to a broader process and because it avoids the possible idea of meanings being somehow deposited in people´s minds.

[3] Just to give an idea of the example. Students agree with the solution of $3x+10=100$ by the teacher; they are (silently) thinking of a scale balance, the teacher is (silently) thinking of an equation (mathematical meaning). When they are asked to solve $3x+100=10$, they say 'that one can't be!'

of that fine reading (Lins et al., 2002; Bueno & Lins, 2002), that is, the intention to produce productive interaction.

How to implement those two points in teacher education projects? Working in Brazil, we have learned how much the structure of teacher education courses ('licenciaturas') — in which the mathematics-pedagogy divide is almost always strong — can pose difficulties for such an implementation. We will consider two possible reasons for that.

First, the mathematical content courses are taught quite as they are taught for future researchers[4], that is, presenting and accepting only *mathematical meanings*[5]; this is clearly insufficient, as we have argued in Lins (in print), because such an approach hides the *differences* in meaning production for mathematics, preventing the future teachers to discuss those differences and the processes involved in their production, something crucial in the reading and decision making processes we have mentioned before.[6]

Efforts have been made to produce mathematics courses that are, in some sense or another, more adequate to the preparation of future teachers. In some cases, as in Cooney et al. (1999), the treatment of the mathematical content is truly innovative, but only elementary topics are treated; in other cases (for instance, Avila, 2001) one has a reduction in the content, but the treatment is much the same as in the 'complete' versions of the course (mathematics of the mathematician). We think that much of the difficulty lies in the understanding of the possible roles for and impact of those (advanced) mathematical courses in the professional practice of teachers, but also in a difficulty to pinpoint what mathematics teachers actually need. In Lins (in print) we outline what the answer to the latter question might be.

Second, and in strong relation to the first point, as soon as a course is organised within a *mathematical discipline* (for instance, Linear Algebra), what the objects treated in it are, is already bound by the relations it has with other objects in that theory. For instance, although 'dimension' can be many things outside Linear Algebra (see Lins et al., 2002), inside Linear Algebra it can only be a few (mathematically equivalent) things. But no matter how much the professor believes his or her students are thinking like him or her, most often then not, they are not; instead of being just an instructional problem if left unattended — students will not learn much, as we pointed before —, this

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[4] We are referring here to 'advanced mathematics' courses, like Calculus, Analysis, Abstract Algebra, Linear Algebra and Metric Spaces and Topology.

[5] More specifically, those are courses on *the mathematics of the mathematician*, not on *the mathematics of the mathematics teacher* (or on *the mathematics of the mathematics educator*).

[6] Also, as much as it happens in school mathematics, not explicitating differences in meaning production leads, quite often, to failure in learning, as we have argued elsewhere. Future mathematics teachers enter university with success stories to tell regarding mathematics, and quite often leave university with failure stories to tell, if not many times scorned by some of their colleagues (not unusually, in Brazil, those who opted for a research career) — as those who failed in school mathematics were also scorned.

would be a superb educational opportunity for future teachers, if acknowledged and dealt with properly, even within those more traditional courses.

But there is a somewhat radical alternative, one that we have studied and discussed in detail in our research group, having developed a framework for its implementation[7]. It consists, first, in adopting a new set of categories to organise the mathematical education of mathematics teachers.[8] Instead of Linear Algebra or Metric Spaces or Geometry, courses are structured around notions such as Space or Measurement or Decision Making. The key idea is that those are everyday categories, well familiar — in their own everyday ways — both to future teachers and to their future students, so they can function as a firm ground from which to proceed, at the same time they are already framing much of what will be present in school mathematics classrooms[9].

Let's consider the 'course' Space, for instance. Beginning from space as this empty place in which things can be (including moving the hands around to refer to it), one can ask questions like: 'Fine, we have this space where things can be, only the space with things in it; this is all we know about it. What changes if we decide to introduce, for instance, a way of comparing the proximity between things? And what if we introduce a system to locate things in this space?' The reader should notice that we begin with a descriptive treatment of space, by dealing with natural and naturalised features of it. But from there we can move much further, and discuss continuity, different coordinate systems, minimal paths, minimal surfaces and more, or we can take those descriptive features and look for other 'spaces' — perhaps totally created, 'artificial' — in which things work similarly to the original one, although they look much different (for instance, as it is done in the book *Flatland*, by E. Abbott[10]).[11]

We were talking about *difference*, further above. In such an environment, one can examine and discuss it in several forms: (i) the difference between *describing* objects (the usual, almost only, approach in everyday life) and *constituting* objects (the only approach in the mathematics of the mathematician; as argued in Lins, in print, and Lins, 2004), that is, the difference between ontological and symbolic objects (cf. Lins, 1992); (ii) the difference between material properties (physical intuition included) of space and mathematical (ideal) properties of space (mathematical intuition included); (iii) the

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[7] As part of the project *A framework for the mathematical courses in mathematics teacher education undergraduate programs*.

[8] Those categories have been used in courses both in our undergraduate program (see, for example, Bueno & Lins, 2002) and postgraduate program (see, for instance, Lins et al., 2002), although the 'titles' of the courses remained the same as before; this was only possible given some considerable degree of teaching freedom found in our institution.

[9] It should be clear that we do not want to suggest that in any country or culture those new categories would be the same, as everyday life is not the same everywhere. This adds a solid and welcome socio-cultural component to curricular design and to teacher education.

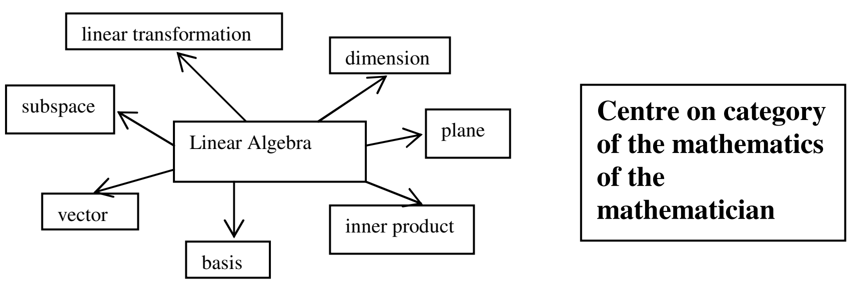
[10] Available online at http://www.alcyone.com/max/lit/flatland/

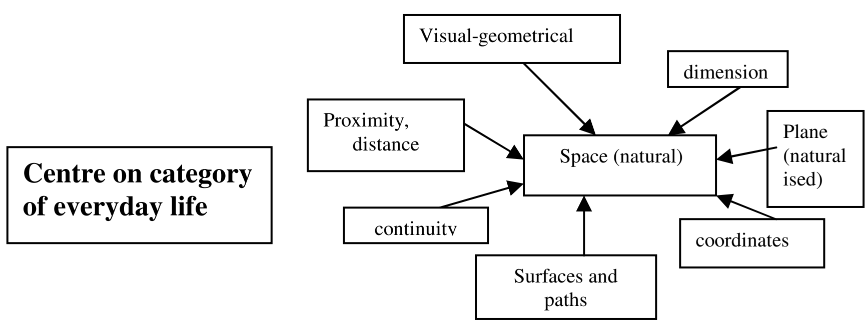
[11] With respect to the new categories and the framework for working within them, the few examplary examples can offer a pointer. Elsewhere the whole design will be presented and discussed (and in more detail than here in the paper presentation, at the conference).

difference between everyday objects (metric straight lines, for instance) and mathematical objects (straight lines as one-dimensional vector subspaces); (iv) the difference between natural (space) and naturalised (plane) objects, and unfamiliar objects (actual infinity), and between monstrous monsters and pet monsters (Lins, 2004).

Moreover, 'pedagogy' and 'mathematics' are not separated — as much as they are not separated in the classrooms.[12] The method through which the investigation and discussion proceeds, generates mathematical and non-mathematical knowledge and understanding *at the same time*, as much as the introduction of mathematical notions also generates mathematical and non-mathematical knowledge and understanding *at the same time*. Pedagogy and mathematics are present as categories for the analysis of the process, but not as categories that actually drive the production of meaning, knowledge and understanding — as much as this also does not happen in actual classrooms. Also, 'advanced' and 'elementary' mathematics are also not separated; one does not come after the other, as the questions and problems that may emerge are not regulated by the pre-established categories (theories) of the mathematics of the mathematician, but by the dynamics of the exploration of the familiar category of Space.

Just to offer a suggestive diagram, summarising the two situations:





Such reflexive experience of *difference* may foster what we had pointed out in points (i) and (ii), at the same time mathematical *maturity* and *understanding* is also

[12] Even as the teacher examines the mathematics produced by the students or their thinking, she is doing so with 'teaching intention.'

fostered. In Lins (in print) we offer some examples of how effective this experience can be, even with beginning teachers, when supported by the theoretical framework that the Model of Semantic Fields offers.

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