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Disciplinary mathematics and school mathematics

WHERE WOULD LEONHARD EULER STAND TODAY AS A SCHOOL TEACHER?

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ABSTRACT

There are two main ways of looking at the relationship between disciplinary and school mathematics. The first is to consider the (desired) impact of disciplinary mathematics on school curricula. The second is to look at the demands of (actual) curricula on teachers and teaching and, as a consequence, the possible approaches to teacher education and professional development.[2] These aspects are not, of course, independent of each other.

One may start by simply considering what disciplinary mathematics has to offer to students and how to input that into curricula, but one soon is led to consider that curricula have lives of their own, be it related, for instance, to tradition (system inertia in relation to change) or to beliefs (research based or not) about how people learn and about mathematics in general (disciplinary, school and so on).

On the other hand, school mathematics is accountable to disciplinary mathematics in a quite precise sense: teaching is a profession in which reforms in professional education rarely remain on the hands of the practitioners themselves, the teachers. Mathematicians[3], mathematics educators[4], general educators, psychologists and a broad range of other professionals are usually in charge of proposing the reforms. I do not wish to argue here whether this is good or bad, simply to state it as a fact. A quite natural consequence of this is that mathematicians tend to have a strong say on 'the mathematics' that is to be 'school mathematics', leading to a visible tension that is possibly resolved by constructs such as Y. Chevallard's *didactical transposition*.

In search of insights on those issues I was led to consider both the status of Leonhard Euler as a mathematician in his lifetime and his mathematical knowledge as seen from the point of view of present-day mathematics.

Although an accomplished mathematician, excellent problem-solver and superbly fluent in applications, it seems that if he were he required to take the tests we give our undergraduates students on Calculus, Analysis, Linear Algebra and Algebra

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[2] In this paper, whenever 'teacher' and 'teaching' are mentioned they refer to 'mathematics teacher' and 'mathematics teaching' unless otherwise explicited.

[3] By 'mathematicians' I mean professionals whose primary activity is to produce research in mathematics. There are places in which mathematics educators will say they *are* mathematicians (possibly in view of their first degree in mathematics), but I prefer to make this explicit distinction. Mathematics educators are not mathematicians, as much as mathematicians are not mathematics educators simply because they, in most cases, are teachers.

[4] Understood *here* as researchers in mathematics education; in general I count school teachers as mathematics educators.

(just to mention a few basic courses), there would be a substantial chance that he would fail them. The reason is quite obvious: he knew nothing of arithmetised (epsilon-delta) Calculus and Analysis, he knew nothing of vector spaces, linear independence, bases and dimension, he knew nothing of groups, rings, fields, his notion of 'function' was that of an analytical expression and he knew nothing of sets. The reason is also obvious: those things did not even exist at his time.

But even failing those tests Euler would probably make a quite good mathematics school teacher.

What can we learn from this?

One could conclude that the kind of mathematics our undergrads (future teachers) study is not *necessary* for them in their profession. One could also conclude that it is not *sufficient*. Or that it is not relevant — echoing Felix Klein's claim.[5]

But one can also be led to further examine what was it that characterised Euler's excellence as a mathematician and a problem-solver *and led to doing so having in mind the profession of teaching mathematics*. That could imply an inversion: instead of seeing those courses as aiming at *content*, seeing them as *means*, *environments* in which students would have the opportunity to develop a sort of 'mathematical fluency'.

Eventually — and this is part of the work of our research group — this may lead to another, stronger, inversion: organising teacher pre-service education on the basis of everyday life categories and, from there, approaching mathematics as a way of framing a particular worldview.

The relevance of such issues to the theme of the Working Group is, I think, to consider the difference between mathematics education understood as *education <u>for</u> mathematics* and as *education <u>through</u> mathematics* and, in view of this difference, to produce further insights on the relations between disciplinary and school mathematics.

In the longer version of this text those and other issues are further discussed.

[5] On the Introduction to his *ElementaryMathematics from an Advanced Standpoint*.