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CHAPTER 3

A BRIEF ESSAY ON THE NEED TO CONSIDER THE "SUPERFICIAL" ASPECTS OF LEARNING ALGEBRA

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INTRODUCTION

There should be no doubt that the mathematical education of a person has to aim at the development of a *deep* understanding of the subject (concepts, techniques, and applications). And this is true in particular for algebra education. Research and development in the field has been, and continues to be, concerned both with producing a sound understanding of teaching and learning processes and with producing the means through which that goal can be achieved.

However, the *superficial* side of the understanding of the subject has been left, I think, considerably unattended. By this I do not mean the mistakes people make when they transfer the *superficial* structure of, say, an algebraic transformation to another situation to which it does not apply. For instance, transferring:

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$$2(a+b)=2a+2b$$

into

$$(a+b)^2=a^2+b^2.$$

With respect to that there has been plenty of documented research, particularly in the 1970s and 80s (see, for instance, Lins, 1992; Lins & Kaput, 2004).

What I do mean is that there are *superficial* aspects of learning mathematics that can and should be explicitly considered, but have not been. A good metaphor here seems to be this: if one dives into a lake, before reaching deep (and indeed even shallow) waters, it is necessary to go through the surface.

Let's consider a child who is learning to speak (a major, major, achievement). If she says, "Daddy, I maked a present for you!" it is highly unlikely that the father will offer any kind of correction for the misconjugated verb. Instead, it is very likely that he will be quite happy to notice that the child is developing a sense of time as related to language use. Is "maked" a sign of deep or of superficial understanding?

I am quite sure that all of us could come up with many similar examples.

Still, with respect to learning to speak a first language, most people do not treat it as if there was a correct or best sequence for mastering its different aspects; assessment of whether or not things are going well is made largely on the basis of how well the child functions socially. And, finally, hardly any person takes sole responsibility for a child learning to speak; brothers and sisters, grandparents and other relatives, neighbours, other children in various situations, TV, radio and so on, are all seen as part of this development. In other words, this is a massively collective enterprise (and thus *deeply* social and cultural).

School education, on the other hand, seems to take a different approach.[1] There are *better* sequences. Correcting *mistakes* is too often considered more important than encouraging new ideas. And even when one adopts a *spiral* curriculum, it is likely that a given teacher will indeed feel s/he is the only one responsible for getting the children adequately through a coil of the spring.

I do not intend to present even a simplified argument against such approach. What I do intend, though, is to take inspiration from how we relate with young children and their development outside school, and to propose that at least to some extent we can benefit from acting similarly in schools. After clarifying what this means I will offer a couple of examples of how it

[1]I am referring to *most* school systems. There are exceptions, but they are so rare as not to make much difference to my argument, unfortunately.

can be implemented in the classroom, arguing that what we propose is different from other, currently available, approaches, which may superficially look similar to it.

FRAMING THE ARGUMENT

Let's look again at the so well documented mistake:

$$(a+b)^2=a^2+b^2$$

Let's view it as the "maked" mentioned above. Doesn't it make sense to say that this could be an encouraging sign that the pupil is perceiving patterns, rather than a discouraging sign that the pupil is *not* understanding that a and b are numbers and that in general that equality does not apply? I think it does, it makes a lot of sense.

Maybe we are driven away from the former view because at the point pupils usually meet such statements we have already told them that letters, in the mathematics context, represent numbers, and so on, so we are *disappointed* that they do not take this into consideration and say.

$$(a+b)^2=a^2+b^2.$$

Take a minute to consider how to factor $x^3 + 1$. If you know the formula by heart (superficial), it will save you time. If you do not, it is possible that you say "hm, that *looks like* $x^3 - 1$ and this one I can handle", and ending up with $x^3 + 1 = x^3 -(-1)$ and so on. That *looks like* is precisely the kind of thing I am interested in, here.

On the other hand, the *mistake* mentioned above happens precisely because

$$2(a+b)=2a+2b$$

does *look like*

$$(a+b)^2=a^2+b^2.$$

Bring the two things together and you will get to my point: we need to educate our pupils' *perception* in algebra, the way they treat the *looks like* factor.

In this paper I will consider two aspects in relation to which this can be done quite early in school. First, the acceptance, as *legitimate* in school mathematics, of expressions involving numbers, letters and arithmetical operations (and eventually the equal sign). Second, the development of a notion of *form* in relation to algebraic expressions, including thinking of algebraic transformations in terms of change of *form*.

I chose these two because I think they are key parts of being fluent in algebraic manipulation. Again, I must remind the reader that I am *not* advocating mindless symbol pushing. I am simply claiming that unless those two aspects are contemplated it is hardly possible to use algebraic language transparently, and that is a big part of what I mean by *being fluent*.

HOW TO EDUCATE PERCEPTION?

A quote taken from Terry Wood, Megan Staples, Sean Larsen and Karen Marrongelle (2008):

> [...] One way to view the differences between mathematics and school mathematics is to describe them as disciplinary practices and learning practices following Cohen and Ball (2001). Consider for example justification and argumentation, these are disciplinary practices in mathematics, but in school mathematics these are learning practices. In mathematics justification and argumentation are disciplinary practices because they are the means by which mathematicians validate new mathematics. In school mathematics argumentation and justification are learning practices because they are the means by which students enhance their understanding of mathematics and their proficiency at doing mathematics.

I think a similar point can be made with respect to algebraic manipulation, but perhaps at a more basic level. In mathematics, algebraic manipulation is a tool that enhances one's ability to justify and argue. Algebraic manipulation happens, so to speak, in the background. In school mathematics, however, too often algebraic manipulation is in the foreground, that is, it is the very subject of study; it is not even a learning practice. How would it be possible to make it into a learning practice, into a *means by which students enhance their understanding of mathematics and their proficiency at doing mathematics*? Pushing it to the background seems a promising approach, as much as justification and argumentation happen, in school mathematics as well as in mathematics, largely in the background.

A second quote, taken from Anne Watson (2008):

> In this paper I argue that school mathematics is not, and perhaps never can be, a subset of the recognised discipline of mathematics, because it has different warrants for truth, different forms of reasoning, different core activities, different purposes, and necessarily truncates mathematical activity. ... The relationship of school mathematics to adult competence is similar to the relationship ... between being made to eat all your spinach and becoming a chef; between being forced to practise scales and becoming a pianist. There are some connections, but they are about having a focus on a narrow subset of semi-fluent expertise in negative social and emotional contexts, without full purpose, context and meaning. That some people become ... beautiful pianists or inspiring cooks is interesting, but what is more interesting is the fact that most people who go through these early experiences do not: instead they merely follow orders, or hate green vegetables, or give up practising their instruments.

This is a quite interesting point: *school mathematics is not, and perhaps never can be, a subset of the recognised discipline of mathematics ... [it] necessarily truncates mathematical activity*. That is, maybe we cannot go too far in making school mathematics—or maybe better, school mathematical activity – look like mathematics or mathematical activity. But as I read about spinach and music a refreshing insight came to my mind. When we want children to learn to wear clothes, are we, to any extent, concerned with whether or not they will become clothing designers? When we want children to learn to eat using cutlery, are we, to any extent, concerned with whether or not they will become gourmets? I don't think so.

Could it be that if we serve spinach to a child early enough we wouldn't come to a point in which the child has to be *made to eat it*? That, instead, eating spinach would simply become something one does? And, again, can't we do so without being concerned at all about the child becoming a chef?

In Brazil, 7th grade (now renamed Year 8, with pupils around 14 years old) is when algebraic manipulation is treated *properly*, that is, becomes a subject of study. Before that, children have very little contact with literal notation in mathematics. To no one's surprise any longer, this is a grade in which the number of pupils failing is significantly higher than in the previous ones. The ever-repeating cry of despair from pupils is "Calculating with letters??". It seems they are being made to eat algebraic expressions and algebraic manipulation, and, of course, the reaction is similar to that of being made to eat spinach. But what if we served them algebraic expressions early, as with the spinach? Could it be that they would get used to them to the extent that they would become natural? And, notice, in doing so no one needs to be interested in whether or not our children are going to become mathematicians.

Yes, I am deliberately trying to be *superficial*.

From Alan Bishop (1994):

> Teacher training in mathematics involves much more than just learning how to manage a classroom effectively. Nor is it just a matter of learning sufficient mathematics to be able to teach that content to school students. Mathematics teachers are passing on values, habits and customs as well as knowledge and skills. They are inducting their students into the culture of mathematics. Culture is not being used here to refer to 'grand' culture, or 'high' culture (as in a 'cultured' person) but merely to reflect the fact that like language, religion or morals, mathematics is part of a culture's store of knowledge, developed by previous (and present) generations and made accessible to succeeding generations. ...

Before we teach children to eat using cutlery do we wait until they *properly* understand what microbes are? No. Before we teach children to use clothes do we wait until they properly understand what social conventions are? What laws are? Do we worry about allowing them to *construct* by themselves the concept of being clothed? No. Do they have to learn the importance of vitamins and iron to our bodies before we can serve them spinach? No, again.

Am I against active learning? No.

But I believe we can and should learn from the way people learn things outside schools. Not the deep aspects of such learning, but the superficial ones. For instance, it seems to be true that using cutlery and using clothes bring a better quality of *social* life for people. And it is for *this* reason that we teach them to our children before doing it makes much sense to them, and not because *potentially* they will be, one day, clothing designers or restaurant owners.[2]

Can we take a similar approach when considering algebraic expressions, for instance? I think we can. The difference might be that we take algebraic expressions into consideration because of something that will only happen in the yet not visible future, that is, learning and using algebra, while using cutlery and clothes and eating spinach are immediately visible as part of children's lives.

As to the title of this section (How to educate *perception*?) my first answer is this: as early as possible. But I would add that two points of view have to be taken into consideration, that of *normal* people and that of mathematics educators (teachers or not).[3]

What is it that *normal* people *see* when they see us *doing algebra*? Calculating with letters. Literally mixing letters with numbers. Mysterious (if not irrational) rules for doing it.

What is it that we, mathematics educators want them to *see*? Legitimate symbolic expressions. Meaningful transformations of those expressions.

I think that an answer to the question in the section title may come from considering both views at the same time: to educate their perception, in this case, means developing *legitimacy* for algebraic expressions and then developing *legitimacy* for expression transformation.

[2]The argument that we dress our children because of, say, the weather, is also interesting, but, again, this is a superficial (immediate). There are native Brazilians, for instance, who live in places where it gets quite cold in the winter, but instead of getting dressed they get used to it.

[3]By *normal* people I mean people who do not have any particular interest in mathematics (as it is the case with mathematics educators, mathematicians, engineers, and so on). *Normal* people have a functional interest in mathematics (everyday life). That means, almost surely, that *normal* people are not algebra users to any extent and, after leaving school (whatever the time they had spent there) gradually algebra is again, slowly or not, reclassified as something akin to ET language. *Normal* people include most children and teenagers.

In the next section I present a possible way to take a *naturalistic* approach in relation to algebraic expressions: if it succeeds, there will be a lot less "*being made to eat spinach*", which will possibly be replaced by "yummies" and "I prefer potatoes". Later in this paper, I extend the argument to the notion of *algebraic form.*

FRUIT SALAD CAN BE GOOD FOR ONE'S HEALTH!

My former PhD supervisor Alan Bell has many times said that representing "3 apples and 2 bananas" by "$3a + 2b$" is *fruit salad algebra,* and that is not a good thing because pupils will not learn that in school algebra letters stand for numbers, not things.

But what if I do not care that, at least at some point, they do not learn that in school algebra letters stand for numbers? Is there still something to be learned from doing fruit salad algebra? This section is an attempt to convince the reader that the answer is a quite important *yes.*

1. *An Activity:* A Weird Snacks and Soda Cans Shop

A shop sells packages of snacks and soda cans. You cannot buy separate snacks or cans in this weird shop... They sell the following packages:

Package A: 1 snack and 3 cans
Package B: 2 snacks and 3 cans
Package C: 2 snacks and 4 cans
Package D: 2 snacks and 6 cans
Package E: 3 snacks and 5 cans
Package F: 4 snacks and 3 cans

How can one buy...

3 snacks and 6 cans?
5 snacks and 8 cans?
1 snack and 1 can?
3 snacks and 9 cans?
(and so on)

To actually find the answers is not hard. It can be made harder with larger numbers or purchases that require multiple combinations. Children will be doing a lot of mental calculations (helpful). They will have to find ways of keeping track of combinations (quite helpful; they may want to make a table with all 36 2-packages combinations. What if I want to represent all 3-packages combinations? Maybe a table won't do, maybe it will...).

But our actual target, when we developed this activity, was something else:

> To buy 3 snacks and 6 cans one buys a package with 1 snack and 3 cans and a package with 2 snacks and 3 cans made into

3 snacks and 6 cans = (1 snack and 3 cans) + (2 snacks and 3 cans)

which easily turns into

3 S and 6 C = (1 S and 3 C) + (2 S and 3 C)

and into

$$3\text{ S} + 6\text{ C} = (1\text{ S} + 3\text{ C}) + (2\text{ S} + 3\text{ C})$$

Junk-food algebra? Well, not a nice name, but in Alan's sense, yes. What might we gain here?

At least two important things, I think. On one hand, pupils are operating with/on expressions *as whole objects*, supported by the *packages* context. On the other hand, literal expressions (such as 3 S + 6 C) are *legitimate* as a notational aid. The abbreviation to S and C may come from the students (sometimes it does), but it may also come as a suggestion by the teacher (not to Vygotsky's opposition).[4] With second to fourth graders mixing letters with numbers with other mathematical signs and no sign of shock or despair, our objectives were reached.

As to the mathematical side of it, we find it hard to see it as anything else than polynomial addition. And in polynomials proper the *letters* are nothing but formal place-markers; (3, 6) instead of 3 S + 6 C would be equally fine. Not so bad: young children writing down and adding polynomials.

And they only had to do it this once to *learn* that doing so is *legitimate* in mathematics.[5]

2. *Another activity:* The Music Shop[6]

Daniel is helping his aunt, who owns a records shop (she went on a boat trip). When he gets to the shop, Monday morning, he re-

[4]We have never witnessed or got a report that a child complained of such abbreviation or said s/he had not *understood* it.

[5]Around 300 children in Brazil, grades 2–4, worked with this activity and were informally post-tested for the persistence of that *legitimacy*, using the activity presented on next section.

[6]Developed at the time when CD's and MP3 players were not available...

alises that he knew his aunt sold the records for a single price, and the tapes for a single price, too. But he forgot to ask what the prices were! Looking around he found a piece of paper with some of Friday's *sales*[7]:

1 record and 5 tapes — R$ 65
3 records and 4 tapes — R$ 85
2 records and 1 tape — R$ 40
4 records and 3 tapes — R$ 90
5 records and 2 tapes — R$ 95

Customers are arriving!! Let's help Daniel to calculate the cost of some new sales!

4 records and 9 tapes
4 records and 2 tapes
3 records and 1 tape
1 record and 1 tape

What is more expensive: a record or a tape?

Teachers suggested the following notation, which pupils readily accepted and used:

$$\begin{array}{rl} & \text{1 record and 5 tapes = R\$ 65} \\ + & \underline{\text{3 records and 4 tapes = R\$ 85}} \\ & \text{4 records and 9 tapes = R\$ 150} \end{array}$$

quickly moving (on their own) to:

$$\begin{array}{rl} & 1\text{ R} + 5\text{ T} = 65 \\ + & \underline{3\text{ R} + 4\text{ T} = 85} \\ & 4\text{ R} + 9\text{ T} = 150 \end{array}$$

What's in it: (i) operating with and on expressions as whole objects; (ii) persistence of the legitimacy of expressions mixing numbers, letters, arithmetical operations and equality sign; and, (iii) more operations with/on expressions (+, –, x, ÷).

In this music algebra Alan's remark holds: the teacher has to make pupils aware that R stands for the price of a record and T for the price of a tape. Also, although it makes sense to write

$$3\text{ R} + 2\text{ T} = (6\text{ R} + 4\text{ T}) \div 2 = 130 \div 2 = 65$$

[7] Notice my didactical emphasis.

it does not make sense to write

$$(4\,R + 9\,T) \div 2 = 2\,R + 4.5\,T = 75$$

because 0.5 of a tape is not of much use. The *logic of the operations* is that of records and tapes, and the arithmetical calculations are only used to produce actual prices of sales. From our theoretical perspective that means there is no algebraic thinking going on (Lins, 1992, 2001).

The teacher may want to ask pupils if they can work out the price of a record and the price of a tape, but by design we always suggest that this is not done. The core of this activity, as in the previous one, is to deal with combinations of expressions.[8]

There are similar activities in which expressions like 4P – 3R are legitimate, and others in which, using decimal numbers, it is less easy to guess individual prices.

THE WATER TANKS

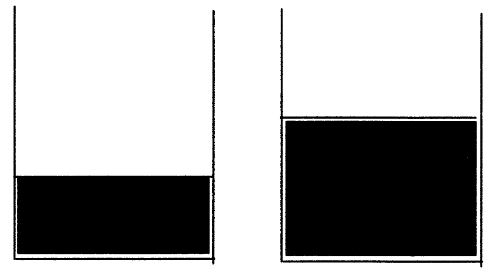
In the previous activities pupils could produce legitimate (for them) literal expressions and operate with/on them. The following activity adds the possibility of developing a sense of form and transformation of such expressions.

These are two identical water tanks. The tank on the left needs another 9 buckets full of water to fill it up. The tank on the right needs another 5 buckets full of water to fill it up. What can we say about this situation?

One can say that "the tank on the right has more water than the tank on the left, just look at the picture". Or that "the tank on the right has more water than the tank on the left, as less water is missing on the right one". Or that "the tank on the right has more water than the tank on the left, as only 5 buckets of water are missing in it, and 9 buckets on the right". From the perspective of our Model of Semantic Fields (Lins, 2001) these three statements *together* with the respective justifications consist in different *knowledge*. The *practical* consequence of including *justification* as a constitutive part of our definition of knowledge is that we can distinguish, in a sufficiently fine and simple way, different pieces of *knowledge* that involve the same statement (proposition).[9]

[8]We have suggested that teachers, in case pupils find out the individual prices, say that's fine but ask them if they can work out the cost of those sales without using them.

[9]We define *knowledge* as "a statement-belief (the statement of something in which a person believes) *together* with a justification that person has for believing in it." This definition immediately and clearly distinguishes the knowledge of a child and a mathematician who both say that " 3 plus 2 is equal to 2 plus three", the child's justification being showing it with her fingers and the mathematician's being that the addition of integers is commutative.



Water Tanks

In this activity we are concerned with the production of legitimate (for the pupils) expressions (statements) and with their justifications for saying so, that is, we are concerned with their *production of knowledge.* Our goal is to get students to operate on those expressions (transform them) first on the basis of a *logic of operations* that has to do with water tanks, water and buckets and then detach, from this *logic*, the transformation rules (*superficially*) produced (that is, in our terms, move it to another semantic field).

Let's agree on calling the amount of water on the left tank X, calling the amount of water on the right tank Y (both 6th grade students' suggestion) and calling the bucket b (teacher's suggestion). A number of statements emerge:

$$X + 4b = Y$$

"because if you *add* 4 buckets on the left there will be only 5 buckets missing"

$$Y - 4b = X$$

"because if you *remove* 4 buckets from the right there will also be 9 buckets missing"

$$X + 2b = Y - 2b$$

"because 7 buckets will be missing on both sides"

$$X + 5b = Y + 1b \text{ (notice: 1b, not b!)}$$

"4 buckets missing on each side"

$$X - 2b = Y - 6b$$

"11 buckets missing on each side"

And then things start to look more interesting:

$$X + 6b = X + 2b$$

"because, because... Oh, no, it's not X, it's Y...!"
Teacher: "What could make that statement true?"
Student: "If the bucket didn't have a bottom!!"

$$X + 20b = Y + 20b$$

"Anything over 5 and 9 buckets will overflow the tanks!!"
Teacher: "X – 50b = Y – 54b"
Students: "It's impossible to do that!! You can see from the drawing..."

$$Y - X = 4b$$

"Because if you *remove* from Y the same amount that's in X..."
But how can one *do* that?

This last statement is of special interest. Y – X = 4b, X + 4b = Y and Y – 4b = X form the core of a whole-part relationship. Nevertheless, although the *logic of operations* of water tanks, water and buckets renders X + 4b = Y and Y – 4b = X immediately clear and understandable, Y – X = 4b requires some extra imagination: how can one remove X of water from Y if one does not know how much water is in X? One possibility is to remove, say, a glass of water from the left then one from the right, until all the water on the left is removed. Tricky.

It is possible the reader will think I am pushing it too far, but I assure you I am not: we have not met a single instance of spontaneous production of Y – X = 4b by pupils, while X + 4b = Y and Y – 4b = X were always in the first two or three offered. But as soon as the teacher presented them with Y – X = 4b they accepted it as true: "the *difference* between Y and X is 4 buckets" (*of course*, just look at the picture).

Quickly pupils produce a big number of expressions that mean something *to them*, are *legitimate for them*, so now we can begin to operate on those expressions *as objects*.

Teacher: "How would you *describe* the direct transformation[10] of
X + 4b = Y into X + 5b = Y + 1b?"
Students: "Add 1 bucket to each tank..."

And from there, with a new *legitimate* action, we could move to generating new legitimate statements offering both "tank-based" and "direct transformation" justifications: Y – 5b = X – 1b either "because 6 buckets missing on each side" or from "remove 1 bucket from each side of Y – 4b = X."

[10] Naming of action.

Finally, let's move to *source, target, transformation*: "Transform X + 4b = Y to make it *look like* b = ...".[11] That was the homework. The next day, there were two kinds of answers:

$$b = \frac{Y - X}{4}$$

and

$$1b = Y - X - 3b$$

There was nothing in the whole process that the pupils were not able to do before it. What have they learned, then? They have *learned* that they may do all that *in mathematics*: legitimacy. They have *learned* that expressions may be directly transformed following rules that legitimately (for them) apply to those (legitimate) expressions. They have *learned* that transforming an expression to give it another, given, form is something people do in some situations (legitimacy).

CLOSING REMARKS

I would like to go back to the metaphor I mentioned at the beginning of this paper: if one dives into a lake, before reaching deep (and indeed even shallow) waters, it is necessary to go through the surface. The surface is not an unwanted feature of the depth. Quite on the contrary, it not only delineates where depth is to be found, it is also *the* access gate to depth.

As I said also at the beginning, yes, algebra education must aim at the appreciation and understanding of deeper structures. But what is wrong with mastering superficialities before mastering deep structure? Nothing, I say. But *mastering superficialities* is not the same as *mastering deep structure*, so the former has to be dealt with in its own terms and, I argue, the *superficial* features of mathematics are more of the order of cultural values than of the order of *subject matter*: it is less a matter of *learning* in the more traditional sense and more a matter of *acceptance*, so, *the earlier, the better*.

Unfortunately, too many people are put off by mathematics. But they are not put off by its depth but rather by its surface. That means, I think, that the failure of our students in algebra is not the failure of *those who tried and failed*. It is rather the failure of *those who never tried to succeed in it* (because what they *see* is not legitimate; it does not make sense).[12]

[11] *Form*: superficial.

[12] Elsewhere (Lins, 1999) I have published a paper in Portuguese, in which I borrow ideas from Monster Theory (Cultural Studies) to further explore this phenomenon.

The approach we have proposed here, with its explicit attention to an early (yet *superficial*) immersion of pupils into superficial features of algebraic activity, aims precisely, in its broader sense, to give them a chance to really face deep structures. If and when that happens, they will be actually able to decide whether or not they like it. They may decide to eat the spinach; they may decide they prefer pizza.

And that will, ultimately, give *teachers* a chance to teach.

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